Réel = $\mathbb{R}$

Dedekind, Lacan, and the Construction of Continuity

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> The real emerges from the impossibilities or inconsistencies in the symbolic.
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> —Lacan, Seminar XVII[1]

> If all points of the straight line fall into two classes such that every point of the first class lies to the left of every point of the second class, then there exists one and only one point which produces this division of all points into two classes.
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> — Richard Dedekind, *Stetigkeit und irrationale Zahlen* (1872)[2]

> *The **Real** is not the void alone, but the persistence of continuity through the void. The **Symbolic** continuously stitches this impossible continuity together, while the **Imaginary** is the phenomenological experience of this continuity as coherent form.*

One of the greatest misunderstandings surrounding Jacques Lacan's notion of the Real is the assumption that the Real designates some hidden empirical substance lying behind language: raw reality before symbolization, traumatic fullness prior to discourse, or an ineffable kernel inaccessible to words. But Lacan repeatedly defines the Real not as hidden fullness, but as *impossibility*.

Yet perhaps *impossibility* itself must be reread: *im-passable.*

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1 "The Real, Really? – LACANONLINE.COM," accessed May 28, 2026, https://www.lacanonline.com/2025/08/the-real-really/.

2 Richard Dedekind, *Stetigkeit und irrationale Zahlen*, with University of Michigan (Braunschweig : F. Vieweg, 1892), http://archive.org/details/stetigkeitundir00dedegoog.

Not simply what cannot be realized, but what cannot be passed over because it is itself the operation of passage. The *impossible* is not merely the obstacle preventing the jump. The impossible is the structural threshold through which continuity becomes possible. In this sense, impossibility may be read almost as impassibility: not what blocks passage from outside, but what cannot itself be passed over because it is the very operation of passage.

This is precisely what appears in Dedekind's construction of the real number. The irrational number is not a hidden point filling a preexisting hole in continuity. It is the cut itself insofar as the cut generates continuity. The gap is productive.

Likewise, the Lacanian Real is not hidden beyond the Symbolic order. The Real emerges where the Symbolic fails to close consistently upon itself, yet nevertheless continues to hold together. The Real is the persistence of continuity through constitutive *impossibility*.

This is why continuity is never simple identity with itself. The Ship of Theseus, the Sorites paradox, the bald man, the heap, and induction puzzles all reveal the same structure: continuity survives not because rupture is absent, but because rupture itself becomes structurally operative.

> The Symbolic stitches this impossible continuity together.
> The Imaginary experiences it as coherent form.
> The Real persists as the *impossibility* through which the entire structure holds.

Dedekind's Discovery of the Gap

Many Lacanians unconsciously preserve this homophonity between *Real* and *reality* and therefore spend some effort explaining that the Real is "not reality"; at least "not reality in the ordinary sense!"[3]

But perhaps Lacan chose the word Réel precisely because mathematics had already transformed its meaning. The clue was in front of our eyes all along:

$$\text{Réel} = \mathbb{R}$$

The Real is not empirical reality. In mathematics, the "real" already designates a continuity constructed through constitutive cuts. Dedekind's real number is not a hidden point filling a gap in the rational line; it is the cut itself insofar as the cut generates continuity. In mathematics, the real appears precisely where continuity becomes thinkable through

[3] NeuroLaunch editorial team, "Lacanian Psychology: Unraveling the Complexities of Jacques Lacan's Theories," *NeuroLaunch.Com*, September 14, 2024, https://neurolaunch.com/lacanian-psychology/.

impossibility. This becomes visible with exceptional clarity in Richard Dedekind’s construction of the real numbers:

The classical intuition imagines irrational numbers as mysterious hidden points inserted into gaps within the rational line. One first imagines continuity, then notices absences such as:

$$\sqrt{2}$$

and finally introduces irrational numbers to repair the incompleteness.

But Dedekind’s construction is far more radical.

$$\sqrt{2}$$

is not first a mysterious hidden point later discovered somewhere between rationals. It is the cut separating:

$$\{q \in \mathbb{Q} \mid q^2 < 2\}$$

from

$$\{q \in \mathbb{Q} \mid q^2 > 2\}$$

The number is therefore:

neither side, nor an external filler, but the operation of division itself. Beautiful, isn’t it?

This changes the ontology of continuity completely. Continuity is no longer something that exists before division and later becomes fractured. Continuity emerges through division itself. The cut is productive.

Réel = $\mathbb{R}$: Lacan and the Mathematical Real

At this point Lacan suddenly becomes mathematically legible. The barred subject:

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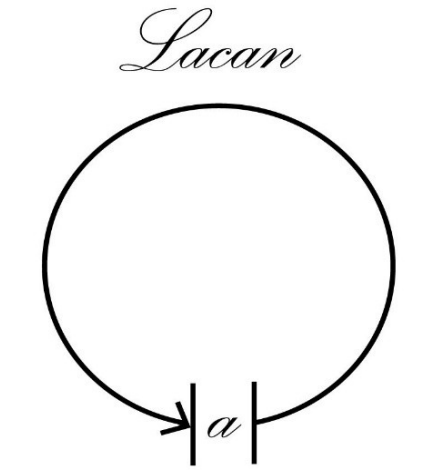
is not a complete subject subsequently wounded by language. The subject does not preexist symbolic division and later become fragmented by signifiers.

The subject is the cut itself insofar as the cut generates a new consistency. Exactly like the irrational number. The subject is neither on one side of the signifier nor the other, neither hidden behind language nor fully present within it. The subject emerges as the impossible consistency generated by symbolic partition.

This is why Lacan insists that *the subject is represented by a signifier for another signifier*.[4] The subject never appears positively as substance. It appears only through division within the symbolic chain.

Likewise, the Lacanian Real is not hidden fullness outside language. The Real is what prevents symbolic closure while simultaneously making symbolic continuity possible.

The Real is not what fills the gap. The Real is the consistency of the gap itself. This also transforms the meaning of continuity: Ordinarily we imagine continuity as the absence of interruption. A line appears continuous because no hole is visible within it. But Dedekind reveals the opposite: continuity is constructed through the void. The cut is not what destroys continuity; it is what allows continuity to become formally thinkable.



The Return of the Object Cause of Desire

The circuit of desire closes only through the cut. *Objet petit a* is not the lost object hidden beyond the symbolic order, but the constitutive gap around which continuity organizes itself. Like a Dedekind cut generating the real number through division, the void here is not an interruption of the circuit but its very condition of persistence and return. The arrow returns because the gap holds. This is the same structure underlying the Sorites paradox and every induction puzzle: at what precise moment does accumulated continuity secretly cross into another order? Which grain creates the

[4] Lacan, Jacques. The Seminar. Book XI. The Four Fundamental Concepts of Psychoanalysis, 1964. Trans. Alan Sheridan. London: Hogarth Press and Institute of Psycho-Analysis, 1977. p. 207; via: “Signifier in Lacanian Psychoanalysis,” No Subject, March 22, 2026, https://nosubject.com/Signifier.

> heap? Which removed hair produces baldness? Which replaced plank transforms the Ship of Theseus into another ship? The paradox emerges because continuity is not the absence of rupture, but the persistence of form through constitutive rupture itself. The circuit of desire does not break at the gap; it is generated by it. Objet petit a functions like a Dedekind cut within continuity: not a missing piece later restored, but the constitutive division through which the circuit remains alive.

The void is therefore not external to continuity. The void is its constitutive element.

This gives an entirely new way of approaching ancient paradoxes of identity and gradual change. Take the Sorites paradox:

At which grain does a heap begin?

At which removed hair does baldness appear?

At which replaced plank does the Ship of Theseus become another ship?

We should “mind the gap.” Yet perhaps we cannot, because the mind is itself the gap.

The point of transformation cannot be fully occupied by consciousness because consciousness itself emerges through the constitutive cut organizing continuity. The transition from quantity to quality, from heap to non-heap, from hair to baldness, from one ship to another, is not hidden behind the process like a secret object waiting to be discovered. The threshold is *im-passable* because it is the very operation through which passage occurs.

The jump cannot observe itself jumping. The subject therefore cannot stand outside the cut in order to map it objectively, because the subject is itself generated through the cut. The mind is not what contemplates the gap from afar.

On the Sorites Paradox and Constitutive Gaps

The paradox emerges because the transformation does not happen through the hairs, grains, or planks themselves, but in the constitutive gaps between them. No individual grain creates the heap, no single removed hair produces baldness, and no final replaced plank suddenly transforms the Ship of Theseus into another ship. The transition occurs in an im-passable threshold that cannot itself be localized positively within the continuity it organizes.

We search for the moment of transformation among the objects, while the transformation takes place in the void structuring their continuity.

This is why the paradox cannot be solved simply by adding more empirical knowledge. The jump from quantity to quality does not occur as another observable element within the chain. It emerges through the constitutive cut holding the chain together while simultaneously preventing it from fully coinciding with itself.

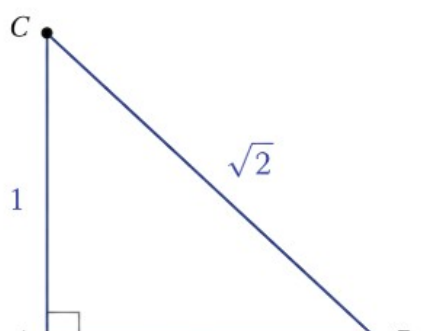
The Real appears precisely there: not as hidden substance behind continuity, but as the impossible consistency generated through the gaps internal to continuity itself.

Classically, this produces logical embarrassment because one searches for the exact point where identity suddenly “jumps.” But perhaps the jump is not accidental. Perhaps continuity itself depends upon an invisible constitutive cut.

The heap is not a stable positive object to which grains are externally added. The heap emerges through a threshold generated by partition. Likewise, baldness is not a positive state suddenly appearing after one decisive hair falls. The identity of “bald” emerges through a constitutive non-coincidence internal to continuity itself. The Ship of Theseus therefore becomes newly legible. The ship does not remain identical because an invisible essence secretly survives beneath material replacement. Nor does it simply become absolutely other after a final plank is exchanged. Rather, identity itself is constructed through an impossible continuity generated by constitutive difference. This sounds very Hegelian indeed. The ship survives through non-identity. Exactly like the subject. Exactly like the irrational number. Exactly like the Lacanian Real.

This is perhaps why Lacan was drawn obsessively toward topology, knots, cuts, Möbius strips, Borromean links, and mathematics in general. Not because mathematics offered decorative metaphors for psychoanalysis, but because modern mathematics had already discovered an ontology in which division generates consistency.

It is only through the Symbolic that we are ever introduced to the Real, because the Real does not exist prior to the cut as some hidden fullness silently waiting behind language. The Real stands out of language in a kind of *Aus-stand*: a German word that simultaneously means “to stand outside” and “to cease functioning.” The Real therefore appears precisely where the Symbolic no longer functions seamlessly, where language stumbles upon its own impossibility while nevertheless continuing to hold continuity together.



The hypotenuse cannot be fully derived from either side alone. It emerges through an im-passable gap internal to their relation itself. The transformation does not occur in the sides as positive quantities,

but in the constitutive structure organizing their continuity. Geometry here repeats the structure of the Sorites paradox and the Ship of Theseus: the transition from one regime to another cannot be localized empirically within the chain of elements themselves. The Real appears as the impossible consistency generated through the distance holding continuity together.

But this exteriority should not be misunderstood metaphysically. The Real is not simply outside language, because without an inside there can be no outside at all. The outside emerges only through the operation of symbolic partition itself. The Real therefore does not precede the Symbolic as primordial fullness; it emerges where the Symbolic fails to coincide fully with itself, yet nevertheless persists, stitches, and returns.

The Real is not beyond the cut. The Real is what stands out through the cut while the cut still holds (=Borromean). And this is why the subject can never coincide fully with itself either. Identity is always and per definition in crisis.